Formüler

Formüllerin doğru olduğu koşullar ve serilerin kapsam alanı belirtilmemiştir. Sağdaki sayı sayfa numarasıdır. Aynı formül birkaç farklı yerde belirebilir.

$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1} < \sqrt{n}$ 38

$10^k > k$ 39

$1 + r + r^2 + \cdots + r^k = \frac{1-r^{k+1}}{1-r}$ 40

$(1+s)^n \geq 1 + ns$ 41

$(1-r)^n \geq 1 - nr$ 41

$1 + px \leq (1+x)^p$ 41

$(n+1)x^n \leq nx^{n+1}$ 42

$\frac{x^p-1}{p} < \frac{x^q-1}{q}$ 43

$1 + px \leq (1+x)^p$ 43

$1 + px \geq (1+x)^p$ 43

$1 - px \leq (1-x)^p$ 44

$2 \leq (1+1/n)^n \leq 3$ 44

$\left(1 + \frac{x}{n}\right)^n < \left(1 + \frac{x}{n+1}\right)^{n+1}$ 44

$(1+k/n)^n \leq [(1+1/n)^n]^k$ 45

$\sqrt{ab} \leq \frac{a+b}{2}$ 45

$\frac{a+bx^4}{x^2} \geq 2(ab)^{1/2}$ 51

$(abc)^{1/3} \leq \frac{a+b+c}{3}$ 52

$(a_1a_2\cdots a_n)^{\frac{1}{n}} \leq \frac{a_1+\cdots+a_n}{n}$ 53

$\sqrt[n+1]{ab^n} \leq \frac{a+nb}{n+1}$ 58

$\left(1 \pm \frac{1}{n}\right)^n \leq \left(1 \pm \frac{1}{1+n}\right)^{n+1}$ 59

$\left(1 + \frac{1}{n+1}\right)^{n+2} < \left(1 + \frac{1}{n}\right)^{n+1}$ 59

$n! < \left(\frac{n+1}{2}\right)^n$ 59

$\frac{2^n n!}{n^n} < \left(\frac{n+1}{n}\right)^n$ 59

$1 \cdot \frac{1}{2^2} \cdot \frac{1}{3^3} \cdot \frac{1}{4^4} \cdots \frac{1}{n^n} < \left(\frac{2}{n+1}\right)^{n(n+1)/2}$ 61

$1^2 \cdot 2^2 \cdot 3^3 \cdot 4^4 \cdots n^n \leq \left(\frac{2n+1}{3}\right)^{n(n+1)/2}$ 61

$s = \sum_{i=1}^n a_i$ ise $\prod_{i=1}^n (1 + a_i) \leq \sum_{i=0}^n \frac{s^i}{i!}$ 62

$\left(1 + \frac{s}{n}\right)^n \leq 1 + \frac{s}{1!} + \frac{s^2}{2!} + \cdots + \frac{s^n}{n!}$ 62

$\sqrt{2}\,\sqrt[4]{4}\,\sqrt[8]{8} \cdots \sqrt[2^n]{2^n} \leq n + 1$ 62

$x_{n+1} = 1 - x_n^2$ 67

$x_{n+1} = \sqrt{x_n}$ 67

$x_{n+1} = -x_n^2 + 3x_n + 1$ 67

$x_{n+1} = 6 - 1/x_n$ 67

$y_{n+1} = \frac{1}{y_n} + 1$ 68

$x_{n+1} = \sqrt{s + x_n}$ 68, 116, 153

$\lim a = a$ 73

$\lim 1/n = 0$ 73

$\lim \frac{1}{n^2} = 0$ 74

$\lim \frac{n-3}{n^2+n-5} = 0$ 74

$\lim r/n^q = 0$ 75

$a_{n+1} = \frac{a}{1+a_n}$ 79

$b_{n+1} = \frac{a}{b_n}$ 79

$\lim n!/n^n = 0$ 82, 117, 163

$\lim \frac{1}{n} \sum_{i=1}^n \frac{1}{i} = 0$ 83

$\lim \frac{\frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{n+n}}{n} = 0$ 83

$\frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{n+n}$ 83

$\lim \left(\sqrt{n^2 + n} - n\right) = 1/2$ 93

$\lim \frac{3n^2-4n+5}{4n^2-5n+1} = \frac{3}{4}$ 94

$\lim_{n\to\infty} \frac{n^p-n^q}{(n+1)^p-(n+1)^q} = 1$ 95

$\lim \left(\frac{1}{n^2+1} + \frac{2}{n^2+2} + \cdots + \frac{n}{n^2+n}\right) = \frac{1}{2}$ 95

$\lim \sum_{i=1}^n \sum_{j=1}^i \frac{j}{n^3} = \frac{1}{6}$ 95

$\lim\ a_n = \lim \frac{a_0+\cdots+a_{n-1}}{n}$ 96

$\lim\ (1+n+n^2)^{1/n} = 1$ 96

$\lim \sum_{k=1}^n \frac{k^2+3k+1}{(k+2)!} = \frac{3}{2}$ 97

$\lim \left(1-\frac{2}{2\cdot 3}\right)\left(1-\frac{2}{3\cdot 4}\right)\left(1-\frac{2}{4\cdot 5}\right)\cdots\left(1-\frac{2}{n(n+1)}\right) = \frac{1}{3}$ 97

$\lim \left(\sqrt{n^2+1}-n\right) = 0$ 97

$\lim \frac{n^k}{n^{k+1}-(n-1)^{k+1}} = \frac{1}{k+1}$ 97

$\lim \sum_{i=1}^n \frac{1}{\sqrt{n^2+i}} = 1$ 98

$\lim \left(\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \cdots + \frac{n}{2^n}\right) = 2$ 98

$\lim \sum_{k=1}^n \frac{k^3+6k^2+11k+5}{(k+3)!}$ 98

$\lim \sum_{i=1}^n \sum_{j=1}^i \frac{j^2}{n^4}$ 98

$\lim\ r^n$ 101

$\lim\ (1+r+r^2+\cdots+r^n)$ 102

$\lim\ r^n/n! = 0$ 103

$\lim\ (r+1/n)^n$ 104

$\lim r^{1/n} = 1$ 104, 136, 201

$\lim a^{x_n}$ 105

$\lim nr^n = 0$ 105, 117, 140

$\lim n^q r^n = 0$ 105, 117, 147

$\lim n^{1/n} = 1$ 105, 137, 182, 201, 291, 315, 316

$\lim \sum_{i=1}^{n-1} \frac{1}{i\cdot(n-i)} = 0$ 105

$a_{n+2} = \frac{a_n+a_{n+1}}{2}$ 106, 146

$\lim \frac{n+4}{3n^2+2} = 0$ 108

$\lim_{n\to\infty} n^{1/n^2} = 1$ 108

$f_{n+2} = f_n + f_{n+1}$ 108, 148

$a_{n+2} = (a_n a_{n+1})^{1/2} \longrightarrow (a_0 a_1^2)^{1/3}$ 108

$\frac{a_0+\cdots+a_{n-1}}{n}$ 111, 122, 232, 288

$na^{n-1}(1-a) < 1-a^n < n(1-a)$ 112

$na^{n-1}(1-a) < 1-a^n < n(1-a)$ 112

$\frac{a_0+\cdots+a_n}{b_0+\cdots+b_n}$ 112

$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}$ 114

$\frac{1}{1^3}+\frac{1}{2^3}+\frac{1}{3^3}+\frac{1}{4^3}+\cdots+\frac{1}{n^3}$ 115

$\frac{1}{1^k}+\frac{1}{2^k}+\frac{1}{3^k}+\frac{1}{4^k}+\cdots+\frac{1}{n^k}$ 115, 125

$2-\frac{1}{n} \geq \frac{1}{1^2}+\frac{1}{2^2}+\cdots+\frac{1}{n^2} \geq \frac{3n}{2n+1}$ 115

$a_{n+1} = \frac{a_n+b_n}{2}$ ve $b_{n+1} = \sqrt{a_n b_n}$ 116, 147

$3a_{n+1} = 2 + a_n^3$ 116, 117

$\lim_{n\to\infty} n^k r^n = 0$ 117

$a_n = \frac{1\cdot3\cdot5\cdots(2n-1)}{2\cdot4\cdot6\cdots(2n)}$ 117

$a_{n+1} = \frac{3(1+a_n)}{3+a_n}$ 117

$\frac{1}{n}+\frac{1}{n+1}+\cdots+\frac{1}{n+n}$ 117

$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$ 117

$a_{n+1} = \frac{a_n+b_n}{2}$ ve $b_{n+1} = \sqrt{a_{n+1} b_n}$ 118

$a_{n+1} = \frac{a_n+b_n}{2}$ ve $a_{n+1}b_{n+1} = a_n b_n$ 118

$x_{n+1} = \frac{s}{1+x_n}$ 118

$x_{n+1} = \frac{s}{x_n} + 1$ 118

$\underbrace{\sqrt{1+\sqrt{1+\sqrt{1+\cdots+\sqrt{1}}}}}_{n \text{ tane}}$ 118

$\frac{1}{1}+\frac{1}{2}+\cdots+\frac{1}{n}$ 119

$\sum_{i=0}^{n} \frac{1}{\sqrt{i+1}+\sqrt{i}} = \sqrt{n+1}$ 122

$\lim \frac{u_0x_0+\cdots+u_nx_n}{u_0+\cdots+u_n} = x_n$ 123

$\lim \frac{x_n}{y_n} = \lim \frac{x_n-x_{n-1}}{y_n-y_{n-1}}$ 124

$\lim \frac{1^k+2^k+\cdots+n^k}{n^{k+1}} = \frac{1}{k+1}$ 124

$|x_k - x_{k+1}| < 1/2^k$ 127

$|x_k - x_{k+1}| < c^k$ 129

$\lim \sqrt[n]{1^2 + 2^2 + \cdots + n^2}$ 137

$a_{2n+1} = \frac{a_{2n}+a_{2n-1}}{2}$ ve $a_{2n+2} = \frac{a_{2n}a_{2n-1}}{a_{2n+1}}$ 137

$x_{n+1} = \frac{x_n+2}{x_n+1}$ 139

$r + 2r^2 + \cdots + nr^n$ 140

$0{,}9999\ldots = 1$ 142

$a_{n+1} = \sqrt{2a_n}$ 146

$a_{n+1} = \frac{2a_nb_n}{a_n+b_n}$ ve $b_{n+1} = \frac{a_n+b_n}{2}$ 147

$a_{n+1} = \frac{b_n+c_n}{2}$, $b_{n+1} = \frac{a_n+c_n}{2}$, $c_{n+1} = \frac{a_n+b_n}{2}$ 147

$a_{n+1} = 1 + \frac{1}{1+a_n} = \frac{2+a_n}{1+a_n}$ 147

$\sqrt{2} = 1 + \frac{1}{2+\frac{1}{2+\frac{1}{2+\cdots}}}$ 147

$a_{n+1} = \sqrt{2 + a_n}$ 147

$\lim\,(\sqrt{n+1} - \sqrt{n})$ 148

$x_{n+1} = \frac{x_n+a}{x_n+1}$ 151

$x_{n+1} = 6 - 1/x_n$ 153

$((1 + 1/n)^n)_{n>0}$ 161

$\lim \left(1 + \frac{1}{n}\right)^n = e$ 162

$\left(1 + \frac{1}{n}\right)^n \leq \left(1 + \frac{1}{n+1}\right)^{n+1} \leq e$ 162

$n! > \left(\frac{n}{e}\right)^n$ 162

$n! \geq \left(\frac{n+1}{e}\right)^n$ 163

$300! > 100^{300}$ 163

$\lim\,(n!)^{1/n} = \infty$ 163, 210, 316

$\left(\left(1 + \frac{x}{n}\right)^n\right)_n$ 164

$\left(1 + \frac{k}{n}\right)^n \leq e^k$ 164

$\left(\left(1 - \frac{x}{n}\right)^n\right)_n$ 165

$\lim\,(1 + q/n)^n = e^q$ 165

$\left(-1+\frac{1}{n}\right)^n$ 167
$e^p \le \lim \left(1+\frac{r}{n}\right)^n \le e^q$ 167
$\left(1+\frac{1}{n}\right)^n \le \sum_{i=0}^n \frac{1}{i!}$ 169
$\sum_{i=0}^n 1/i!$ 170
$\sum_{i=0}^n x^i/i!$ 170
$\exp x = \lim \left(1+\frac{x}{1!}+\frac{x^2}{2!}+\cdots+\frac{x^n}{n!}\right)$ 173, 178
$(1+x_0)(1+x_1)\cdots(1+x_n) \le \exp(x_0+\cdots+x_n)$ 173
$\lim \sum_{i=0}^n 1/i! = e$ 174
$\exp x = \lim \left(1+\frac{x}{n}\right)^n$ 174
$(x-a_1)\cdots(x-a_k) \ge x^k - (a_1+\cdots+a_k)\, x^{k-1}$ 176
$0 \le \frac{1}{i!} - \binom{n}{i}\frac{1}{n^i} \le \frac{1}{2n}\frac{1}{(i-2)!}$ 176
$\lim n(n^{1/n}-1) = \infty$ 178
$\left|\exp x - \sum_{i=0}^n \frac{x^i}{i!}\right| \le \frac{|x|^{n+1}}{(n+1)!}\exp|x|$ 180
$\lim\ (n+1)^{1/n}$ 182
$\lim\ (n!)^{1/(n-1)!}$ 182
$\lim e^{1/n} = 1$ 182
$\lim n(e^{1/n}-1) = 1$ 182
$\exp(x+y) = \exp x \exp y$ 182
$\exp 0 = 1$ 187
$\exp(-x) = \frac{1}{\exp x}$ 187
$\exp x < \exp y$ 188
$f(n) = [e^{(n-1)/2}]+1$ 193
$(n+1)^{\frac{1}{n+1}} < n^{\frac{1}{n}}$ 201
$(n+1)^n \le n^{n+1}$ 202
$\lim \left(1+\frac{1}{n^2}\right)^n = 1.$ 202
$\lim \left(1+\frac{1}{n}\right)^{n^2} = \infty$ 202
$\lim \left(\frac{n-1}{1+n}\right)^n = \frac{1}{e^2}$ 202
$\lim \left(\frac{3n-5}{7+4n}\right)^n = 0$ 203

$\lim \left(\frac{3n-5}{7+3n}\right)^n = e^{-4}$ 203

$a_{k+1} = \sqrt{6 + a_k}$ 203

$\sum_{i=1}^n 1/i$ 205

$\lim \frac{n^3-2n+7}{3n^2+n+4} = \infty$ 209

$\sum \frac{x^{2^i}}{1-x^{2^{i+1}}}$ 236

$\sum \frac{1}{i(i+1)} = 1$ 238

$\sum r^i = \frac{1}{1-r}$ 239, 304, 312, 321

$\sum \frac{1}{i^2}$ 242, 335, 338

$\sum \frac{2i+3}{i(i+1)(i+2)(i+3)} = \frac{1}{3}$ 244

$\sum \frac{1}{i(i+2)} = \frac{3}{4}$ 244

$\sum(\sqrt{i+1} - \sqrt{i})$ 244, 251

$\lim_{n\to\infty}(1 + a^n)^n = 1$ 251

$\lim_{n\to\infty}(n^{1/n} - 1)^{1/n} = 1$ 252

$\sum \frac{2i+1}{i^2(i+1)^2} = 1$ 244

$\sum \frac{1}{i(i+2)(i+3)} = \frac{13}{36}$ 244

$\sum \frac{a_n}{\prod_{i=0}^n (1+a_i)} = 1 - \lim \frac{1}{\prod_{i=0}^n (1+a_i)}$ 245

$\sum \frac{i-1}{i!} = 1$ 246

$\sum \frac{1}{i(i+1)(i+3)} = \frac{7}{36}$ 246

$\sum \frac{3i+1}{(i+1)(i+2)(i+3)}$ 247

$\frac{1}{1\cdot 3} + \frac{1}{2\cdot 4} + \frac{1}{3\cdot 5} + \frac{1}{4\cdot 6} + \cdots = \frac{3}{2}$ 247

$\sum \frac{1}{i(i+1)(i+2)} = \frac{1}{4}$ 247

$\sum_i \frac{1}{i(k+i)} = \frac{1}{k}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k}\right)$ 247

$\sum x^i/i!$ 248, 304, 312, 321

$\sum_{i=1}^\infty 1/i$ 248, 251

$\sum \frac{x_{i+1}-x_i}{x_i} = \infty$ 248

$\sum \frac{x_{i+1}-x_i}{x_{i+1}} = \infty$ 248

$\sum x_i$ yakınsaksa, $\lim \sum_{i=n}^{2n} x_i = 0$ 249

$\lim\left(\frac{1}{n^2}+\frac{1}{(n+1)^2}+\cdots+\frac{1}{(n+n)^2}\right)=0$ 249

$\lim_{n\to\infty} nx_n = 0$ 251

$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots=\frac{\pi^2}{6}$ 253

$1-\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{6}-\frac{1}{8}+\frac{1}{5}-\frac{1}{10}-\frac{1}{12}+\cdots$ 259

$\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+\cdots$ 260

$\sum_n\left(\sum_{i=1}^{i-1}\frac{1}{i(n-i)}\right)=\infty$ 267

$\sum\frac{x_{i+1}-x_i}{x_i^{p-1}x_{i+1}}$ 267

$\sum_{n\in C}\frac{1}{n}$ 269

$\frac{1}{1\times3}+\frac{1}{5\times7}+\frac{1}{9\times11}+\frac{1}{13\times15}+\cdots$ 270

$\sum x^i/i$ 270, 273

$\sum\frac{2i}{3i^3+4}$ 270

$\sum\frac{1}{\sqrt{i^2-1}}$ 271

$\sum\frac{1}{\sqrt{i^2+1}}$ 271

$\sum(i^{1/i}-1)$ 272

$\sum\frac{2i}{3i^3-4}$ 272

$\sum\frac{2^i+3^{i+1}}{i^2+7^{2i+1}}$ 273

$\sum\sqrt{x_ix_{i+1}}$ 273

$\sum\frac{1}{n^p-n^q}$ 273, 332

$\sum_i\frac{1}{i(i+1)\cdots(i+k)}$ 274

$\sum\frac{1}{1+1/2+\cdots+1/i}=\infty$ 274

$(\exp x)(\exp y)=\exp(x+y)$ 285

$\sum(n+1)r^n=\frac{1}{(1-r)^2}$ 285

$\sum(-1)^n\left(\sum_{i=1}^{n+1}\frac{1}{i(n+1-i)}\right)$ 285

$\sin(x+y)=\sin x\cos y+\sin y\cos x$ 286

$\cos(x+y)=\cos x\cos y-\sin x\sin y$ 286

$\sum ir^i=\frac{r}{(1-r)^2}$ 286, 326

$\sin^2 x+\cos^2 x=1$ 286

$\lim \frac{1}{n} \sum_{i=1}^{n} \frac{1}{i} = 0$ 289

$\lim \frac{1}{n} \sum_{i=1}^{n} \sqrt[i]{i} = 1$ 289

$\lim\, (x_1 \cdots x_n)^{1/n} = \lim x_n$ 290

$\lim \frac{n}{\sqrt[n]{n!}} = e$ 291

$1 - \frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \frac{4}{5} - \frac{5}{6} + \cdots$ 298

$1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \frac{1}{9} + \frac{1}{11} - \frac{1}{6} + \cdots$ 301

$\sum_i i^k x^i$ 304

$\sum (-5)^i / 3^{2i+1}(i+1)$ 304

$\sum i!/15^i$ 305

$\sum_i \binom{i+k}{i} x^i$ 305

$\sum \frac{1 \cdot 4 \cdot 7 \cdots (3n+1)}{(n+1)!} x^n$ 307

$(1+x)^n = \sum \frac{n(n-1)\cdots(n-i+1)}{i!} x^i$ 308

$(1+x)^\alpha = \sum \frac{\alpha(\alpha-1)\cdots(\alpha-i+1)}{i!} x^i$ 308

$\sum \frac{x^{2^{k-1}}}{1-x^{2^k}}$ 309

$\sum 2^i (i!)^2/(2i)!$ 309

$\sum i/a^i$ 309

$\cosh x = \sum x^{2i}/(2i)!$ 311

$\sinh x = \sum x^{2i+1}/(2i+1)!$ 311

$\sum_{i=0}^{n} \binom{2n+1}{2i} = 2^n$ 311

$\cosh(2x) = 2\, \cosh x\, \sinh x$ 311

$x^i/i!$ 312

$\sum 1/i^{5i}$ 312

$\sum \left(\frac{3i-1}{2i+1}\right)^{5i} x^i$ 312

$\sum 1/i^{i/2}$ 313

$\lim_{n\to\infty} \frac{n}{(n!)^{1/n}} = e$ 315

$\sum \frac{X^i}{i!}$ 321

$\sum i! X^i$ 321

$\sum_i 1/i^p$ 327

$\sum \frac{1}{i\sqrt{i+1}}$ 330

$\sum 1/\sqrt{i(i^2-1)}$ 330

$\sum \frac{i}{i^3+1}$ 330

$\sum_i 1/(2i+1)^p$ 330

$\sum_i (-1)^i/i^p$ 332

$\sum 2^i x_{2^i}$ 333

$\sum (k_{i+1}-k_i)x_{k_i}$ 334

$\sum \frac{(2i)!}{4^i(i+1)!i!}$ 335

$\sum \frac{(2i)!}{4^i(i+1)!\,i!}$ 337

$\lim \frac{(2n)!}{4^n(n+1)!\,n!} = 0$ 337

$\sum \frac{1\cdot 4\cdot 7\cdots(3i+1)}{(i+1)!3^i}$ 338

$1 + \frac{1+a}{1+b} + \frac{(1+a)(2+a)}{(1+b)(2+b)} + \frac{(1+a)(2+a)(3+a)}{(1+b)(2+b)(3+b)} + \cdots$ 338

$\sum_i \frac{a(a+c)(a+2c)\cdots(a+ic)}{b(b+d)(b+2d)\cdots(b+id)}$ 338

$1 + \frac{a}{1}\,\frac{b}{c}x + \frac{a(a+1)}{1\cdot 2}\,\frac{b(b+1)}{c(c+1)}x^2 + \frac{a(a+1)(a+2)}{1\cdot 2\cdot 3}\,\frac{b(b+1)(b+2)}{c(c+1)(c+2)}x^3 + \cdots$ 339

$\sum_n \frac{e^{inz}}{n}$ 344

$|a^x - 1| \leq |x|\,|a-1|$ 354

$\lim\,[nx]/n = x$ 355

$\prod_{i=2}^{\infty}(1-1/i)$ 366

$\sum y_i \leq \prod(1+y_i) \leq \exp\left(\sum y_i\right)$ 366

$\prod(1+1/i^s)$ 367

$\prod \frac{4i^2}{(2i-1)(2i+1)}$ 367

$\prod(1+z^n)$ 369

$\prod\left(1-\frac{(-1)^i}{i}\right) = 1$ 369

$\prod\left(1-\frac{z^2}{i^2}\right)$ 369

$\prod\left(1-\frac{1}{i^2}\right) = \frac{1}{2}$ 369

$\prod(1-1/i^s)$ 370

$\prod_{i=2}^{\infty}\left(1-\frac{2}{i(i+2)}\right) = \frac{1}{3}$ 370